

Worcester County Mathematics League

Freshman Meet 3

March 13, 2019

COACHES' COPY

ROUNDS, ANSWERS, AND SOLUTIONS

WORCESTER COUNTY MATHEMATICS LEAGUE



Freshman Meet 3 – March 13th 2019

Round 1: Graphing on a Number Line

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Graph the following inequality on the number line provided.

$$18 > 6 - 3x > -24$$

2. Graph the following inequality on the number line provided

$$2\frac{1}{2} - \frac{x+1}{3} < x - \frac{x}{4}$$

3. Graph the following inequality on the number line provided.

$$\left| \frac{3}{x} + 2 \right| > 5$$

ANSWERS

(1 pt.) 1. 

(2 pt.) 2. 

(3 pt.) 3. 

WORCESTER COUNTY MATHEMATICS LEAGUE



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Round 2: Operations on Polynomials

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. What is the quotient when $(6x^{3m} + 25x^{2m} + 21x^m - 10)$ by $(2x^m + 5)$

2. Simplify and write the answer as a polynomial in standard form:

$$(5x + 3)^2 - (5x - 3)^2 + (5x)^2 + 3^2$$

3. Solve for all values of x

$$(2x + 3)(6x + 1) = (2x + 3)(5x - 7)$$

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

WORCESTER COUNTY MATHEMATICS LEAGUE



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Round 3: Techniques of Counting and Probability

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. In a cooler, there are 3 colas, 1 ginger ale, 4 cherry sodas, and 4 root beers in the cooler. Without looking in the cooler, what is the probability of pulling out a cola?
2. How many different ways can a ship's signal sender arrange 5 red flags, 3 white flags, and 6 blue flags, using all of the flags each time?
3. Betsy has 8 dresses and 5 skirts. In how many ways can she arrange 4 dresses and 2 skirts on a rack which will only hold 6 hangers?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

WORCESTER COUNTY MATHEMATICS LEAGUE



Freshman Meet 3 – March 13th 2019

Round 4: Perimeter, Area, and Volume

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Ignoring units the perimeter of a square equals the surface area of a cube. Find the perimeter of the square?
2. The base of a right rectangular prism has a length of 5 inches and a perimeter of 18 inches. What is the volume of the prism, in cubic inches, given its surface area is 166 square inches?
3. A circle is inscribed in a square. Four points of the circle are on the square. What percent of the area of the square is outside the circle? (note: use 3.14 for pi)

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____



Freshman Meet 3 – March 13th 2019

TEAM ROUND

All answers must be in simplest exact form in the answer section. (3 points each)

NO CALCULATOR ALLOWED

1. Simplify the expression:

$$\frac{x^4 - x}{x^3 - x}$$

2. On a map, each $\frac{5}{8}$ inch represents 15 miles. If two cities are 10 inches apart on a map. How many miles separate the cities?
3. A tribe in New Guinea has eleven letters in their alphabet. The letters A, B, E, G, I, K, O, P, R, T, and U. Suppose license plates of five letters utilize only the letters used by this tribe, How many license plates of five letters are possible that begin with G or K, end with the letter T, and have no letters that repeat?
4. Convert 311_5 to base 7
5. The area of a rectangle is 216 cm^2 . Find the perimeter length and width if their ratio is 3:2.
6. A man travels to his destination at 20 mph and returns at 30mph. What is his average speed for the trip in miles per hour?
7. Graph the following inequality on the number line provided.

$$5 + 2|3(x - 2) + 5| \geq 12$$

8. Factor completely:

$$x^3 + 5x^2 - 4x - 20$$

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TEAM ROUND ANSWER SHEET

1. _____

2. _____

3. _____

4. _____

5. Length = _____ Width = _____

6. _____

7. 

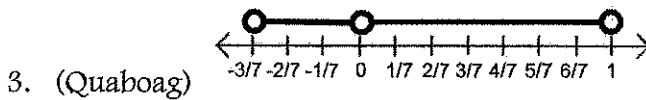
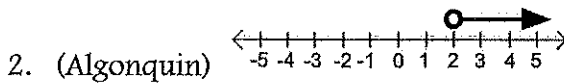
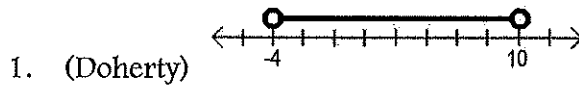
8. _____



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Answer Key

Round 1:



Round 2:

1. $3x^{2m} + 5x^m - 2$ (Auburn)
2. $25x^2 + 60x + 9$ (Tahanto)
3. $x = -1.5$ or $-3/2$, $x = -8$ (Assabet Valley)

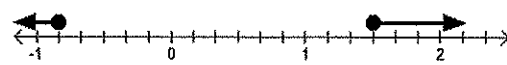
Round 3:

1. $1/4$ or $.25$ or 25% (Notre Dame)
2. $168,168$ (Algonquin)
3. $504,000$ (Oxford)

Round 4:

1. $8/3$ (St. Johns)
2. 140 (Quaboag)
3. $.215$ or 21.5% (St. Peter Marian)

Team Round:

$(X^2 + X + 1) / (X + 1)$ 1. (Worcester Academy)	5. Length = 18, Width = 12 (Bancroft)
2. 240 (Bartlett)	6. 24 (Tantasqua)
3. 1008 (Hopedale)	7. (Hopedale)
	
4. 1447 (Bromfield)	8. $(x+2)(x-2)(x+5)$ (Hudson)

Freshman Meet 3 – SOLUTIONS
Round 1: Graphing on a Number Line

1. Graph the following inequality on the number line provided

$$18 > 6 - 3x > -24$$

Solution:

$$18 > 6 - 3x > -24$$

Subtract 6

$$12 > -3x > -30$$

$$\frac{12}{-3} > \frac{3x}{-3} > \frac{-30}{-3}$$

$$-4 < x < 10$$

Solution:



2. Graph the following inequality on the number line provided.

$$2\frac{1}{2} - \frac{x+1}{3} < x - \frac{x}{4}$$

Solution:

$$\frac{5}{2} - \frac{x+1}{3} < x - \frac{x}{4}$$

Multiple by 12 LCD

$$30 - 4(x+1) < 12x - 3x$$

$$30 - 4x - 4 < 9x$$

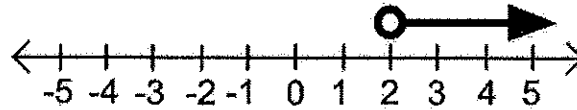
$$26 - 4x < 9x$$

$$26 < 13x$$

$$2 < x$$



Solution:



3. Graph the following inequality on the number line provided.

$$\left| \frac{3}{x} + 2 \right| > 5$$

Solution:

$$\left| \frac{3}{x} + 2 \right| > 5$$

$$\frac{3}{x} + 2 > 5 \quad \text{OR} \quad \frac{3}{x} + 2 < -5$$

$$\frac{3}{x} > 3 \quad \text{OR} \quad \frac{3}{x} < -7$$

$$3 > 3x \quad \text{OR} \quad 3 < -7x$$

$$1 > x \quad \text{OR} \quad -\frac{3}{7} < x$$

$$-\frac{3}{7} < x < 0 \quad \text{OR} \quad 0 < x < 1$$

However x can't be zero

Solution:





Freshman Meet 3 – SOLUTIONS
Round 2: Operations on Polynomials

1. What is the quotient when $(6x^{3m} + 25x^{2m} + 21x^m - 10)$ by $(2x^m + 5)$

Solution:

$$\begin{array}{r} 3x^{2m} + 5x^m - 2 \\ 2x^m + 5 \overline{) 6x^{3m} + 25x^{2m} + 21x^m - 10} \\ \underline{-6x^{3m} + -15x^{2m}} \\ 10x^{2m} + 21x^m \\ \underline{-10x^{2m} - 25x^m} \\ \phantom{10x^{2m}} -4x^m - 10 \\ \underline{-4x^m - 10} \\ \phantom{10x^{2m}} 0 \end{array}$$

Solution: $3x^{2m} + 5x^m - 2$

1. Simplify and write the answer as a polynomial in standard form:

$$(5x + 3)^2 - (5x - 3)^2 + (5x)^2 + 3^2$$

Solution :

$$\begin{aligned} & [(5x + 3)^2 - (5x - 3)^2] + (5x)^2 + 3^2 \\ & [((5x + 3) - (5x - 3))((5x + 3) + (5x - 3)) + 25x^2 + 9] \\ & [6(10x)] + 25x^2 + 9 \\ & 60x + 25x^2 + 9 \end{aligned}$$

Solution: $25x^2 + 60x + 9$



3 Solve for all values of x

$$(2x + 3)(6x + 1) = (2x + 3)(5x - 7)$$

Solution:

$$(2x + 3)(6x + 1) - (2x + 3)(5x - 7) = 0$$

$$(2x + 3)(6x + 1 - 5x + 7) = 0$$

$$(2x + 3)(x + 8) = 0$$

$$x = -1.5 \text{ or } -3/2, x = -8$$

Solution: $x = -1.5$ or $-3/2, x = -8$



Freshman Meet 3 – SOLUTIONS

Round 3: Techniques of Counting and Probability

1. In a cooler, there are 3 colas, 1 ginger ale, 4 cherry sodas, and 4 root beers in the cooler. Without looking in the cooler, what is the probability of pulling out a cola?

Solution:

3 bottles of cola / 12 total bottles

Solution: $1/4$ or 0.25 or 25%

1. How many different ways can a ship's signal sender arrange 5 red flags, 3 white flags, and 6 blue flags, using all of the flags each time?

Solution:

$$\frac{14!}{5! * 6! * 3!}$$
$$\frac{(14)(13)(12)(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)(6)(5)(4)(3)(2)(1)(3)(2)(1)}$$

Solution: 168,168



2. Betsy has 8 dresses and 5 skirts. In how many ways can she arrange 4 dresses and 2 skirts on a rack which will only hold 6 hangers?

Solution:

$$\begin{array}{l} \text{Dresses} \\ \frac{8 * 7 * 6 * 5}{1 * 2 * 3 * 4} = 70 \end{array}$$

$$\begin{array}{l} \text{Skirts} \\ \frac{5 * 4}{1 * 2} = 10 \end{array}$$

Ways of selecting 6 garments

$$70 * 10 = 700$$

Places on the Rack

$$6 * 5 * 4 * 3 * 2 * 1 = 720$$

Total Arrangements

$$720 * 700 = 504,000$$

Solution: 504,000



Freshman Meet 3 – SOLUTIONS

Round 4: Parameter, Area, and Volume

1. Ignoring units the perimeter of a square equals the surface area of a cube. Find the exact perimeter of the square?

Solution:

$$4x = 6x^2$$

$$0 = 6x^2 - 4x$$

$$0 = 2x(3x - 2)$$

$$x = 2/3$$

Therefore: $P = 4 * 2/3 = 8/3$

Solution: $8/3$

2. The base of a right rectangular prism has a length of 5 inches and a perimeter of 18 inches. What is the volume of the prism, in cubic inches, given its surface area is 166 square inches?

Solution:

$$\text{Perimeter of Base} = 2l + 2w$$

$$18 = (2 * 5) + (2 * w)$$

$$8 = 2 * w$$

$$w = 4$$

$$\text{Surface Area} = 2(5 * 4) + 2(4h) + 2(5h)$$

$$166 = 40 + 8h + 10h$$

$$126 = 18h$$

$$h = 7$$

$$v = l * h * w$$

$$v = 5 * 4 * 7$$



$$\text{Volume} = 140 \text{ in}^3$$

Solution: 140

3. A circle is inscribed in a square. Four points of the circle are on the square. What percent of the area of the square is outside the circle? (note: use 3.14 for pi)

Solution:

Example – Radius of circle is 2, so square side is 4

$$\text{area of square} = 4 * 4 = 16$$

$$\text{area of circle} = 3.14 * 2 * 2 = 12.56$$

$$16 - 12.56 = 3.44$$

$$\frac{3.44}{16} = .215 = 21.5\%$$

Solution: 0.215 or 21.5%

WORCESTER COUNTY MATHEMATICS LEAGUE



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TEAM ROUND - SOLUTIONS

1. Simplify the expression:

$$\frac{x^4 - x}{x^3 - x}$$

Solution:

$$\frac{x^4 - x}{x^3 - x} = \frac{x(x^3 - 1)}{x(x^2 - 1)} = \frac{x^3 - 1}{x^2 - 1} = \frac{(x - 1)(x^2 + x + 1)}{(x - 1)(x + 1)}$$

$$\text{Solution: } \frac{(x^2 + x + 1)}{(x + 1)}$$

2. On a map, each $\frac{5}{8}$ inch represents 15 miles. If two cities are 10 inches apart on a map. How many miles separate the cities?

Solution:

$$\frac{5}{8} = 15/x$$

$$\frac{5}{8} * x = 150$$

$$x = 240 \text{ miles}$$

Solution: 240

3. A tribe in New Guinea has eleven letters in their alphabet. The letters A, B, E, G, I, K, O, P, R, T, and U. Suppose license plates of five letters utilize only the letters used by this tribe, How many license plates of five letters are possible that begin with G or K, end with the letter T, and have no letters that repeat?



Solution:

$$G/K \quad _ _ _ T$$

$$2 * 9 * 8 * 7 * 1$$

Solution: 1008 possible combination

4. Convert 311_5 to base 7

Solution:

Change to base 10 first

$$1 * 1 = 1$$

$$1 * 5 = 5$$

$$3 * 25 = 75$$

$$75 + 5 + 1 = 81$$

Then change to base 7

$$\frac{81}{7} = 11 R4, \text{ therefore } 4 = \text{LSB}$$

$$\frac{11}{7} = 1 R4$$

$$\frac{1}{7} = 0 R1, \text{ therefore } 1 = \text{MSB}$$

Solution: 144_7

5. The area of a rectangle is 216 cm^2 . Find the perimeter length and width if their ratio is 3:2.



Solution:

$$3x * 2x = 216$$

$$6x^2 = 216$$

$$x^2 = 36$$

$$x = 6 \text{ or } -6$$

Since lengths have to be positive

$$\text{length} = 3 * 6 = 18$$

$$\text{width} = 2 * 6 = 12$$

Solution: Length = 18, Width = 12

6. A man travels to his destination at 20 mph and returns at 30mph. What is his average speed for the trip in miles per hour?

Solution:

$$T = D/R$$

$$T(\text{going}) = x/20$$

$$T(\text{return}) = x/30$$

$$T(\text{total}) = \frac{x}{20} + \frac{x}{30} = \frac{5x}{60} = \frac{x}{12}$$

$$\text{average rate} = \frac{\text{total distance}}{\text{total time}}$$

$$\frac{x + x}{x/12} = 24 \text{ mph}$$

Solution: 24



7. Graph the following inequality on the number line provided.

$$5 + 2|3(x - 2) + 5| \geq 12$$

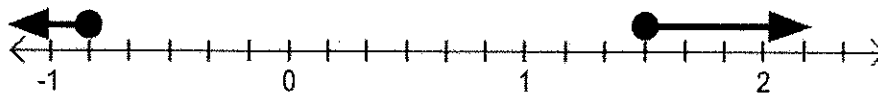
Solution:

$$\begin{aligned} 5 + 2|3x - 6 + 5| &\geq 12 \\ 5 + 2(3x - 1) &\geq 12 \text{ OR } 5 + 2(-3x + 1) \geq 12 \\ 5 + 6x - 2 &\geq 12 \text{ OR } 5 - 6x + 2 \geq 12 \\ 6x + 3 &\geq 12 \text{ OR } -6x + 7 \geq 12 \end{aligned}$$

$$6x \geq 9 \text{ OR } -6x \geq 5$$

$$x \geq \frac{3}{2}, 1.5 \text{ OR } x \leq -\frac{5}{6}$$

Solution:



8. Factor completely:

$$x^3 + 5x^2 - 4x - 20$$

Solution:

$$(x^3 + 5x^2) - (4x + 20)$$

$$x^2(x + 5) - 4(x + 5)$$

$$(x^2 - 4)(x + 5)$$

$$(x + 2)(x - 2)(x + 5)$$

Solution: $(x + 2)(x - 2)(x + 5)$